$$\frac{M'l + M''(l-a)}{M}\frac{f}{l} = \frac{W}{M}.$$

Hence,

$$\frac{P}{M} = \frac{M'l + M''(l-a)}{M}\frac{(f-f')}{l}$$

and

$$\frac{P}{W} = \frac{f-f'}{f}.$$

This proves the second part.

294. Proposed by EMMA GIBSON, Student at Drury College.

A sphere, revolving about a diameter and not acted on by any extraneous force, expands symmetrically; prove that its vis viva varies inversely as its moment of inertia about its diameter.

SOLUTION BY E. B. WILSON, Massachusetts Institute of Technology.

The moment of momentum of the sphere is $I\omega$, where I is the moment of inertia and ω the angular velocity about the axis. This is constant as no external forces are acting. The kinetic energy is $\frac{1}{2}I\omega^2$ or $I^2\omega^2/2I$, which proves the proposition.

MECHANICS.

295. Proposed by B. F. FINKEL, Drury College.

A homogeneous hollow cylinder, whose inner radius is half of its outer radius, rolls without slipping down a plane inclined at an angle α to the horizon. Find its acceleration.

I. SOLUTION BY A. M. HARDING, University of Arkansas.

The external forces acting are W pounds at the center vertically downward, the reaction normal to the plane, and the friction up the plane.

Let R denote the resultant of the last two and let β denote the angle that its direction makes with the normal. Then the equation of motion of the mass center is

$$W\frac{d^2s}{dt^2} = Wg\sin\alpha - R\sin\beta. \qquad (1)$$

If the length of the outer radius of the cylinder is a, then

$$I\frac{d^2\theta}{dt^2} = R\,x\,a\sin\beta = Ra\sin\beta,$$

where the moment of inertia of the cylinder about its axis is $I = \frac{5}{8}Wa^2$.

But, since the cylinder does not slide,

$$s = a\theta. \quad \therefore \frac{d^2s}{dt^2} = a\frac{d^2\theta}{dt^2}.$$